Section 2.4 (page 137)

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (5x - 8)^4$	$u = 5x - 8$	$y = u^4$
3.	$y = \sqrt{x^3 - 7}$	$u = x^3 - 7$	$y = \sqrt{u}$
5.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$

7. $12(4x - 1)^2$ **9.** $-108(4 - 9x)^3$

11. $\frac{1}{2}(5 - t)^{-1/2}(-1) = -1/(2\sqrt{5 - t})$

13. $\frac{1}{3}(6x^2 + 1)^{-2/3}(12x) = 4x/\sqrt[3]{(6x^2 + 1)^2}$

15. $\frac{1}{2}(9 - x^2)^{-3/4}(-2x) = -x/\sqrt[4]{(9 - x^2)^3}$ **17.** $-1/(x - 2)^2$

19. $-2(t - 3)^{-3}(1) = -2/(t - 3)^3$ **21.** $-1/\left[2\sqrt{(x + 2)^3}\right]$

23. $x^2[4(x - 2)^3(1)] + (x - 2)^4(2x) = 2x(x - 2)^3(3x - 2)$

25. $x\left(\frac{1}{2}\right)(1 - x^2)^{-1/2}(-2x) + (1 - x^2)^{1/2}(1) = \dfrac{1 - 2x^2}{\sqrt{1 - x^2}}$

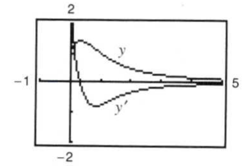
27. $\dfrac{(x^2 + 1)^{1/2}(1) - x(1/2)(x^2 + 1)^{-1/2}(2x)}{x^2 + 1} = \dfrac{1}{\sqrt{(x^2 + 1)^3}}$

29. $\dfrac{-2(x + 5)(x^2 + 10x - 2)}{(x^2 + 2)^3}$ **31.** $\dfrac{-9(1 - 2v)^2}{(v + 1)^4}$

33. $2((x^2 + 3)^5 + x)(5(x^2 + 3)^4(2x) + 1)$
$= 20x(x^2 + 3)^9 + 2(x^2 + 3)^5 + 20x^2(x^2 + 3)^4 + 2x$

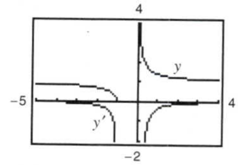
35. $\dfrac{1}{2}(2 + (2 + x^{1/2})^{1/2})^{-1/2}\left(\dfrac{1}{2}(2 + x^{1/2})^{-1/2}\right)\left(\dfrac{1}{2}x^{-1/2}\right)$
$= \dfrac{1}{8\sqrt{x}\left(\sqrt{2 + \sqrt{x}}\right)\left(\sqrt{2 + \sqrt{2 + \sqrt{x}}}\right)}$

37. $(1 - 3x^2 - 4x^{3/2})/[2\sqrt{x}(x^2 + 1)^2]$



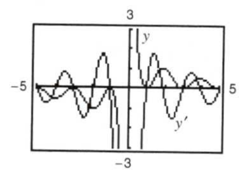
The zero of y' corresponds to the point on the graph of the function where the tangent line is horizontal.

39. $-\dfrac{\sqrt{\dfrac{x + 1}{x}}}{2x(x + 1)}$



y' has no zeros.

41. $-[\pi x \sin(\pi x) + \cos(\pi x) + 1]/x^2$



The zeros of y' correspond to the points on the graph of the function where the tangent lines are horizontal.

43. (a) 1 (b) 2; The slope of $\sin ax$ at the origin is a.

45. $-4 \sin 4x$ **47.** $15 \sec^2 3x$ **49.** $2\pi^2 x \cos(\pi x)^2$

51. $2 \cos 4x$ **53.** $(-1 - \cos^2 x)/\sin^3 x$

55. $8 \sec^2 x \tan x$ **57.** $10 \tan 5\theta \sec^2 5\theta$

59. $\sin 2\theta \cos 2\theta = \dfrac{1}{2} \sin 4\theta$

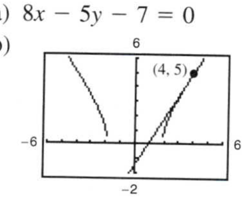
61. $\dfrac{6\pi \sin(\pi t - 1)}{\cos^3(\pi t - 1)}$ **63.** $\dfrac{1}{2\sqrt{x}} + 2x \cos(2x)^2$

65. $2 \sec^2 2x \cos(\tan 2x)$

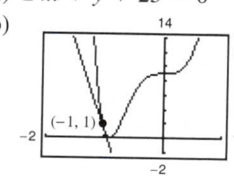
67. $s'(t) = \dfrac{t + 3}{\sqrt{t^2 + 6t - 2}}, \dfrac{6}{5}$ **69.** $f'(x) = \dfrac{-15x^2}{(x^3 - 2)^2}, -\dfrac{3}{5}$

71. $f'(t) = \dfrac{-5}{(t - 1)^2}, -5$ **73.** $y' = -12 \sec^3 4x \tan 4x, 0$

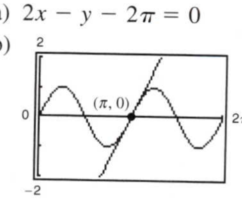
75. (a) $8x - 5y - 7 = 0$
(b)



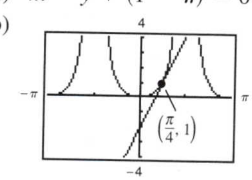
77. (a) $24x + y + 23 = 0$
(b)



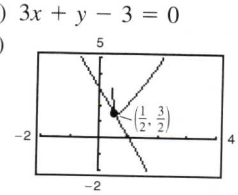
79. (a) $2x - y - 2\pi = 0$
(b)



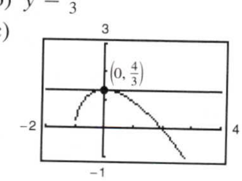
81. (a) $4x - y + (1 - \pi) = 0$
(b)



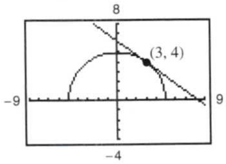
83. (a) $g'(1/2) = -3$
(b) $3x + y - 3 = 0$
(c)



85. (a) $s'(0) = 0$
(b) $y = \frac{4}{3}$
(c)



87. $3x + 4y - 25 = 0$



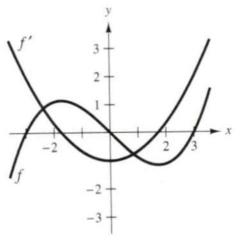
89. $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right), \left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right), \left(\frac{3\pi}{2}, 0\right)$ **91.** $2940(2 - 7x)^2$

93. $\dfrac{2}{(x - 6)^3}$

95. $2(\cos x^2 - 2x^2 \sin x^2)$ **97.** $h''(x) = 18x + 6, 24$

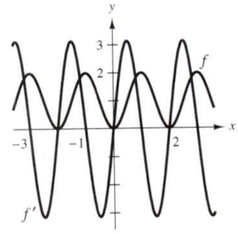
99. $f''(x) = -4x^2 \cos(x^2) - 2\sin(x^2), 0$

101.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

103.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

105. The rate of change of g is three times as fast as the rate of change of f.

107. (a) $g'(x) = f'(x)$ (b) $h'(x) = 2f'(x)$
(c) $r'(x) = -3f'(-3x)$ (d) $s'(x) = f'(x + 2)$

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

109. (a) $\frac{1}{2}$
(b) $s'(5)$ does not exist because g is not differentiable at 6.

111. (a) 1.461 (b) −1.016

113. 0.2 rad, 1.45 rad/sec **115.** 0.04224 cm/sec

117. (a) $x = -1.637t^3 + 19.31t^2 - 0.5t - 1$
(b) $\dfrac{dC}{dt} = -294.66t^2 + 2317.2t - 30$
(c) Because x, the number of units produced in t hours, is not a linear function, and therefore the cost with respect to time t is not linear.

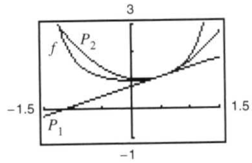
119. (a) Yes, if $f(x + p) = f(x)$ for all x, then $f'(x + p) = f'(x)$, which shows that f' is periodic as well.
(b) Yes, if $g(x) = f(2x)$, then $g'(x) = 2f'(2x)$. Because f' is periodic, so is g'.

121. (a) 0
(b) $f'(x) = 2 \sec x \cdot \sec x \tan x = 2 \sec^2 x \tan x$
$g'(x) = 2 \tan x \sec^2 x = 2 \sec^2 x \tan x$
$f'(x) = g'(x)$

123. Proof **125.** $f'(x) = 2x\left(\dfrac{x^2 - 9}{|x^2 - 9|}\right), x \neq \pm 3$

127. $f'(x) = \cos x \sin x/|\sin x|, x \neq k\pi$

129. (a) $P_1(x) = 2/3(x - \pi/6) + 2/\sqrt{3}$
$P_2(x) = 5/(3\sqrt{3})(x - \pi/6)^2 + 2/3(x - \pi/6) + 2/\sqrt{3}$
(b)



(c) P_2
(d) The accuracy worsens as you move away from $x = \pi/6$.

131. False. If $f(x) = \sin^2 2x$, then $f'(x) = 2(\sin 2x)(2 \cos 2x)$.

133. Putnam Problem A1, 1967